Sinergias Educativas

Article

Didactic proposal to determine the acceleration of gravity

Propuesta didáctica para determinar la aceleración de la gravedad

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Abstract

This paper proposes an alternative didactic methodology to determine the acceleration of gravity using a smart phone inside a box with a steel base and sliding on an inclined plane with a steel base. The experiment is performed by placing oil on the steel surface of the inclined plane, then the smartphone is placed on the box and released from the top of the inclined plane and with the help of the Phyphox application the angle of inclination of the plane and the acceleration of the box is measured. The process is repeated by increasing the angle of inclination and in each case the acceleration of the box is measured. Although no direct proportionality was found between the box acceleration and the sine of the tilt angle, a linear

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Sinergias educativas
October - December Vol. 8 - 4- 2023
http://sinergiaseducativas.mx/index.php/revista/
eISSN: 2661-6661
revistasinergias@soyuo.mx
Page 1-13
Received: June 11 , 2023
Approved: January 12 , 2023

relationship was observed, mainly influenced by the friction between the two steel surfaces.

Keywords: Angle of inclination, acceleration, acceleration of gravity, alternative methodology, theoretical model.

RESUMEN

Este artículo propone una metodología didáctica alternativa para determinar la aceleración de la gravedad utilizando un teléfono inteligente que se encuentra en el interior de una caja con base de acero y que se desliza sobre un plano inclinado con base de acero. Se realiza el experimento colocando aceite sobre la superficie de acero del plano inclinado, luego el teléfono inteligente se coloca sobre la caja y se suelta desde la parte superior del plano inclinado y con ayuda de la aplicación Phyphox se mide el ángulo de inclinación del plano y la aceleración de la caja. Se repite el proceso aumentando el ángulo de inclinación y en cada caso se mide la aceleración de la caja. Aunque no se encontró una proporcionalidad directa entre la aceleración de la caja y el seno del ángulo de inclinación, se observó una relación lineal, influenciada principalmente por el rozamiento existente entre las dos superficies de acero.

Palabras clave: Ángulo de inclinación, aceleración, aceleración de la gravedad, metodología alternativa, modelo teórico.

Introduction

The acceleration of the Earth's gravity is the acceleration that a body undergoes in interaction with the gravitational field of the planet. The acceleration of gravity discovered by Galileo Galilei in 1604 and the formulation of the law of universal gravitation proposed by Sir Isaac Newton in 1687 provided the basis for what is known as the acceleration of gravity and its relation to bodies. For Suwanpayak et al., (2018) at different points on the planet the acceleration of gravity tends to vary, this is mainly due to the Earth's gravitational field.

For that the experimental determination of the acceleration of gravity in different locations has been an object of study with respect to the fallible methods that can be used and that approximate the standard gravity or gravity at sea level. It should be noted that the experimental method, which, through Galileo Galieli, proclaimed by certain historians as the father of experimental physics, Papp, (1961) argues that:

"Galileo does not use experience to find the law, he invokes it only to verify it, already found by deductive reasoning. One does not err by overemphasizing this characteristic of the Galilean method ... The revolutionary methodological innovation introduced by the great Italian in science ... does not consist in the apotheosis of experience with contempt for deductive speculation, but, as we have said, in the masterly synthesis of the triple method of his precursors - philosophical, mathematical and empirical - in one and indivisible unity."(Papp, 1961)

Experimental methods must count on the elements that Galileo traversed in his indivisible unity, which demonstrates the true sense and purpose of experimental research. Related to the determination of the acceleration of gravity, it is evident the postulation and implementation of several methods that supported in the mathematical component and supported in the theory are used to determine the acceleration of gravity, for example, the **principle of free fall** refers to the use of an intelligent timer to detect the time t for a metal ball falling between a point 1 to a distant point 2, the difference between these heights is known as distance h. In contrast, the **simple pendulum method** takes into consideration the principle of simple harmonic motion, where a metal ball of mass m, connected to a string of length L, and using a digital timer to detect the time of oscillation of the ball of 10 revolutions with five repetitions by modifying the angle between 0° and 10°. On the other hand, the **rigid pendulum technique** consists of a rigid body undergoing a fixed-axis rotation around a fixed point. In this experiment, a 100 cm long metal ruler with holes for the rotation points is used, in addition, a digital stopwatch was used to detect the time taken for the metal ruler to complete 10 revolutions, which was estimated from five repetitions, in 5 cm phases, from 5 to 95 cm. In addition, **Atwood's machine** consists of a pulley with two weights of unequal mass and such difference produces a net force which accelerates both hanging masses, and the time between the distance from a point 1 to a distant point 2 is taken. (Suwanpayak et al., 2018).

The methods mentioned above have a mathematical analysis component, which provides experimental values that can be compared with the theoretical value and determine their uncertainty and percentage error.

In addition, there are other types of methods that make use of smartphones or smartphones, for example, through the use of applications I can use the principle of free fall because these devices through the use of sensors, allow to observe, measure and record the data that are being taken. (Kuhn & Vogt, 2013)..

Likewise Martínez Pérez, (2015) mentions about the use of these mobile devices for the determination of the acceleration of gravity by means of the simple pendulum:

... ICTs are equipped with sensors such as accelerometers, gyroscopes, barometers, etc. and, in the last two years, several researches have shown their feasibility to be used in experiences inside the physics classroom or in your laboratory, in acoustic, optical and mechanical phenomena.... (Martínez Pérez, 2015)

The relevance of the aforementioned research highlights the emphasis needed in science teaching. Teachers must structure and deepen concepts, demonstrating the relationship between physical phenomena and variables (Martínez-Borreguero et al., 2018)..

In physical science teaching, the question of how to transmit knowledge to students arises. It is essential to structure methodologies that encourage the development of skills through experience.

At present it does not seem to make sense to speak of the experimental method, but rather of an experimental activity that is part of a body of knowledge, and includes a diversity of methods. Science seeks theories that effectively solve problems, which may be empirical or conceptual; the progress of science seems to occur to the extent that more problems are solved or eluded. In this sense, changes are gradual, accepting the coexistence of rival programs, and although there is a bidirectional relationship between theory and methods, progress in each field may not be simultaneous (Andrés Z. et al., 2006)..

In conclusion, it has been observed that there are multiple methods used to determine the acceleration of gravity, as well as the evolution of this measurement has been evidenced, since it comes from using timers or stopwatches to use smartphones with sensors that allow data collection, the latter is the case of this research, which seeks to determine the acceleration of gravity by placing an object in a plane tilting based on its resultant force.

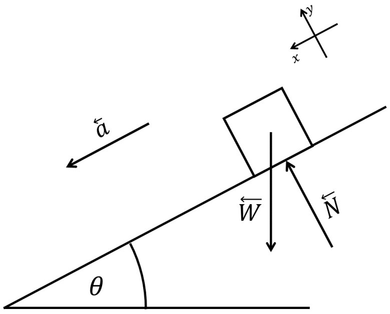
The fundamentals of dynamics were set out in Galileo Galilei's book "*Dialogo sopra i due massimi sistemi del mondo*" which means "*Dialogue on the two main systems of the world*", where the notion of the concept of inertia was implicit and explained. In addition, Galileo's experiments with inclined planes had made it possible to establish mathematical relationships between the kinematic variables for motion with constant velocity and acceleration. (Galilei, 1632). On the other hand, Isaac Newton in his book "Philosophiae naturalis principia mathematica" which means "Mathematical principles of natural philosophy", mentions his three laws of which we will mention two that will be used in the didactic strategy proposed...:

Newton's First Law states "All bodies persevere in their state of rest or uniform motion in a straight line, unless they are forced to change that state by an external force".

Newton's second law: "The change of motion is proportional to the impressed motive force, and is in the same direction as the straight line in which that force is impressed" (Newton, 1686). (Newton, 1686).

Both laws we apply it to the box moving along the inclined plane of angle $\theta$and with the indicated reference system, as shown in figure 1.

Figure 1. Vector description of the forces acting on the box and the reference system used.



Considering the experimental didactic proposal, there are two forces acting on the object, the force of Normal $\left(\vec{N}\right)$ force exerted by the inclined plane on the box and the weight $\left(\vec{W}\right)$ which is the gravitational force exerted by the earth on the box, both forces measured in Newtons, as shown in figure 1. $\vec{W}$ as $\overrightarrow{W_X}\ y\ \overrightarrow{W_Y}$ respectively, according to the indicated reference system, as shown in equations 1 and 2:

$$W_X = W\cos\theta \text{ (Equation 1)}$$

$$W_Y = W \operatorname{sen}\theta \text{ (Equation 2)}$$

In addition, it is observed that Newton's First Law is fulfilled on the vertical axis and Newton's Second Law is fulfilled on the horizontal axis. $\left(\sum \vec{F}\right)$ measured in Newton will be zero, as shown in equation 3.

$$\sum \vec{F} = \vec{0} \text{ (Equation 3)}$$

Considering the reference frame of figure 1 we have equation 4:

$$N - W_Y = 0 \text{ (Equation 4)}$$

Where, the vertical component of the weight (W_Y)measured in Newton, is equal to the product of the mass (m) of the box including the smartphone measured in kilograms and the cosine of the tilt angle

(θ). Therefore, replacing the above in equation 4, we have equation 5, as shown below:

$$N - mg\cos\theta = 0 \text{ (Equation 5)}$$

Now analyzing the horizontal axis, it is observed that Newton's Second Law is fulfilled, so that the sum of forces is equal to mass times acceleration, as shown in Equation 6:

$$\left(\sum \vec{F} = m\vec{a}\right)\text{(Equation 6)}$$

Considering the reference frame of figure 1, we have equation 7:

$$W_X = ma\text{(Equation 7)}$$

Where, the horizontal component of the weight (W_X)measured in Newton, is equal to the product of the mass (m) of the box including the smartphone measured in kilograms and the sine of the angle of inclination (θ). Therefore, replacing the above in equation 7, we have equation 8, as shown below:

$$mg\,\mathrm{sen}\,\theta = ma \text{ (Equation 8)}$$

Analyzing equations 5 and 8, it follows that to determine the acceleration of the box including the smartphone, only equation 8 is needed, since the friction is neglected, therefore, simplifying the mass m of this equation, equation 9 is obtained:

$$a = g\,\mathrm{sen}\,\theta \text{ (Equation 9)}$$

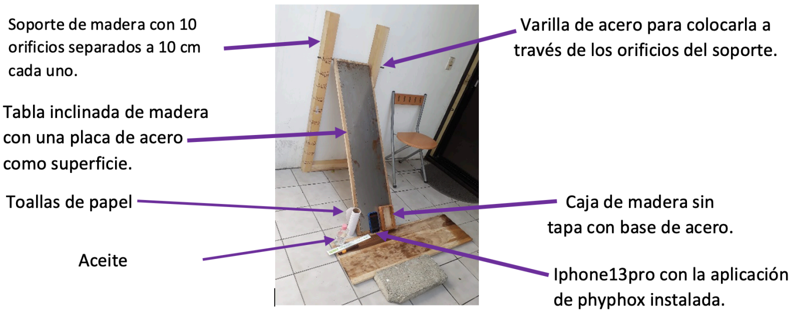
From the obtained result it is observed that the acceleration of the box including the smartphone is directly proportional to the sine of the angle of inclination, therefore, if the sine of the angle of inclination increases in magnitude, the acceleration of the box including the smartphone will increase in the same proportion; which allows obtaining a theoretical model that can be verified with the experimental model obtained in practice. If we compare it with the equation of direct proportion we notice that in the vertical axis goes the acceleration of the box including the included smartphone, in the horizontal axis goes the sine of the angle of inclination and the slope of the graph would be the acceleration of gravity. Then, our didactic experimental proposal consists of increasing the sine of the angle of inclination and then obtaining the acceleration of the box with the smartphone included. In this didactic experimental proposal, the sine of the inclination angle is expected to be directly proportional to the acceleration of the box including the smartphone while sliding down the inclined plane, i.e., if the angle of inclination is increased, the acceleration of the box including the smartphone will increase proportionally because increasing the angle will increase its speed.

LIST OF MATERIALS AND EQUIPMENT

- 1 inclined plane made with:
- 2 wooden boards of length (100,0± 0,1)cm and width (30.0± 0,1)cm;

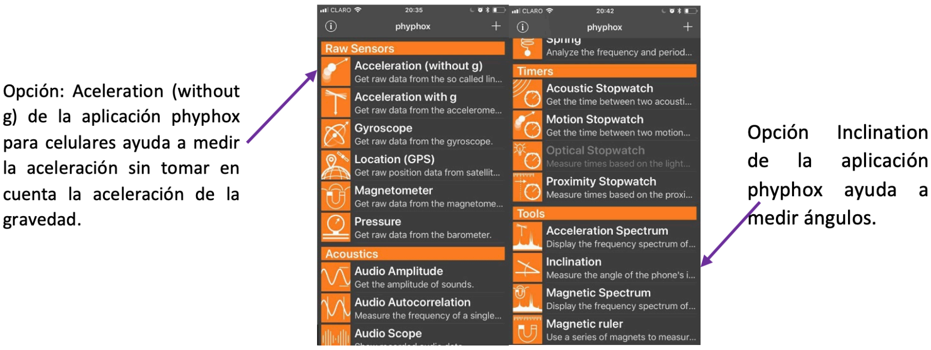
- ☐ 1 steel plate of length (100.0± 0,1)cm and width (30.0± 0,1)cm;
- ☐ 2 hinges
- ☐ 1 wooden stand height (130,0± 0,1)cm and width (50.0± 0,1)cm; with 10 separate holes (10.0± 0,1)cm
- ☐ 1 steel rod length (50.0± 0,1)cm
- ☐ 1 wooden box without lid length (15,0±0.1) cm; width (8.0±0,1) cm and height (3,0±0.1) cm and its base is placed a steel plate of (15.0 0 0.1) cm and width (8.0 0.1) cm.±0.1) cm and width (8.0±0.1 cm
- ☐ 1 ruler of (30.0±0,1)cm
- ☐ 1 brush
- ☐ 1 Paper towels
- ☐ 1 1 liter cooking oil
- ☐ 2 smartphones preferably of the latest generation with the highest number of sensors inside with Phyphox application installed.
- ☐ 1 laptop computer.
- ☐ 1 pencil.

Figure 2. Materials and equipment used in the experiment.



Prepared by the author.

Figure 3. Phyphox application used in the experimentation.



Opción: Aceleration (without g) de la aplicación phyphox para celulares ayuda a medir la aceleración sin tomar en cuenta la aceleración de la gravedad.

Opción Inclination de la aplicación phyphox ayuda a medir ángulos.

Materials and methods

First of all, the inclined plane is assembled with the above-mentioned materials: wooden boards, steel plate, hinges, wooden support, steel rod. Then, with the materials wood box and steel surface, the box is completed. Then with the wooden support and the rod, the inclined plane is placed with a certain angle of inclination, as shown in Figure 1. After them, the smart phone is placed inside the box with the Phyphox application to measure the angle of inclination, in this case it starts with the angle of $(11.66\pm0.01)°$. After that with a brush oil is placed on the entire surface of the inclined plane and on the surface of the box, to reduce the friction effect. Then the laptop is synchronized with the waxed Phyphox application that will measure the acceleration, for this both the smartphone and laptop must be connected to the same wifi network. Now place the smartphone inside the box, then place it on the top of the inclined plane and indicate a mark with the pencil on the front of the box to put it back in the same position for the next measurements. Finally the box is released and the measurement of the acceleration of the box is activated from the laptop with the smartphone while sliding on the inclined plane, the measurement is deactivated when the box reaches the bottom of the inclined plane. The process is repeated 3 times with the same angle of inclination to measure the acceleration, to reduce random errors. After that the angle of inclination will be increased different from the initial one by selecting 7 different values and the whole process is repeated measuring in each case 3 times the acceleration of the box.

Results

The raw data obtained in the experiment are shown below.

Table 1: *Raw data of tilt angle (°) vs. acceleration of the enclosed smartphone case (a)*

Angle of inclination θ/° $\pm\Delta\boldsymbol{\theta} = \pm\mathbf{0,01}°$	Acceleration of the case including the smart phone a/ms-2 $\pm\Delta\boldsymbol{a} = \pm\mathbf{0,01}\boldsymbol{ms}^{-2}$		
11.66°	0,3	0,16	0,2
16.23°	1,11	0,99	0,82
21.13°	1,52	1,51	1,63
26.07°	2,73	2,14	2,47
31.29°	3,31	3,39	3,81
37.08°	4,53	4,26	4,03
42.67°	5,03	5,01	5,1
49.33°	5,81	5,79	5,9

When performing the analysis of the accelerations measurements, it can be observed that there are groups of data for the same inclination angle that have low precision and other groups of data that have high precision, reason for which it was decided to select from each group of accelerations, the 2 most precise ones. In the case of the first group of acceleration data, we will obtain the mean and the uncertainty, as shown in equations 6 and 7 as follows:

$\bar{a} = \frac{0,16 + 0,20}{2} = 0,18\ ms^{-2}$ (Equation 6)

$\Delta\bar{a} = \frac{0,20 - 0,16}{2} = 0,02\ ms^{-2}$ (Equation 7)

This process will be repeated with each of the groups of accelerations found in the data table. Now we will obtain the uncertainty of sinθ , considering the first angle with its respective uncertainty according to equation 8:

$\Delta\,\mathrm{sen}\,\theta = \Delta\theta \cdot \cos\theta = 0,01(\cos 11,66) = 0,0098$ (Equation 8)

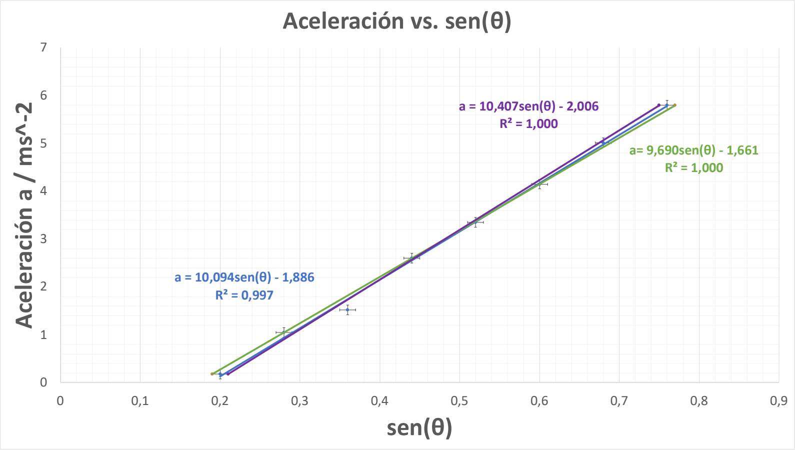
This process is repeated for each of the tilt angles. The table of the processed values is shown below:

Table 2. *Processed data of plane tilt angle and average acceleration of the case including the smartphone with their respective uncertainties.*

Angle of inclination θ/° $\pm\Delta\boldsymbol{\theta} = \pm\mathbf{0,01}°$	Sine of tilt angle $\mathbf{sen}\,\boldsymbol{\theta}$	Uncertainty of the sine of angleθ $\boldsymbol{\Delta\theta}(\mathbf{cos}\,\boldsymbol{\theta})$	Average acceleration $\bar{\boldsymbol{a}}/\boldsymbol{ms}^{-2}$	Average acceleration uncertainty $\pm\Delta\bar{\boldsymbol{a}}/\boldsymbol{ms}^{-2}$
11,66°	0,20	0,01	0,18	0,02
16,23°	0,28	0,01	1,05	0,06
21,13°	0,36	0,01	1,52	0,01
26,07°	0,44	0,01	2,60	0,13
31.29°	0,52	0,01	3,35	0,04
37,08°	0,60	0,01	4,15	0,12
42,67°	0,68	0,01	5,02	0,01
49,33°	0,76	0,01	5,80	0,01

Next, we proceed to construct the block mass graph including the coins as a function of their submerged height. The uncertainties of the measurements will be included in the graph, the line of best fit, the line of maximum and minimum slope to determine the uncertainty of the slope, as shown in Figure 4:

Figure 4. Plot of the acceleration of the box including the smartphone as a function of the sine of the tilt angle, with its uncertainties, the line of best fit, the lines of maximum and minimum slope.



From the graph it can be observed that the uncertainties of the vertical and horizontal axis are small with respect to the scale used. When plotting the line of best fit it can be observed that there is a linear behavior between both variables, since the line does not pass through the origin, this may be an indication of some type of systematic error. Additionally, the line of best fit passes through most of the experimental points, however, there are indications of random errors. As already mentioned in the theoretical framework, the slope (p) in Figure 4 is the acceleration of gravity (g), therefore, equation 9 is obtained:

$p = g$ (Equation 9)

From the graph we obtain the value of the slope, in this case:

$p = g = 10{,}094\ ms^{-2}$ (Equation 10)

With the help of the line of maximum and minimum slope, the uncertainty is obtained. Δpin this case the value is:

$$\Delta g = 0{,}225\ ms^{-2}$$

Therefore, the acceleration of gravity with its uncertainty is

$$g = (10{,}094 \pm 0{,}225) ms^{-2}$$

Discussion

According to the initial hypothesis and the results obtained in the graph, it can be concluded that the hypothesis was not fulfilled, since it was shown that the relationship between the acceleration of the box including the smartphone and the sine of the angle of inclination has a linear behavior and not a direct proportion. A possible reason why experimentally a direct proportionality between both variables was not obtained is that there is a small friction between the two contact surfaces since there are possibilities that the oil has not filled the imperfections of both surfaces. Based on graph 4, it can be seen that the correlation coefficient (R2) is 0.997, which means that there is a strong linear relationship between the acceleration of the case including the smartphone and the sine of the tilt angle. The value obtained for the acceleration of gravity using the didactic methodological proposal was $(10{,}094 \pm 0{,}225) ms^{-2}$a value very close to that found by (Garcia et al., 2008) where the acceleration of gravity was (9.796 ± 0.004) m.s-2. It can also be observed that its accuracy is high, since the percentage of error was 2.952%, additionally the range of theoretical values is within and very close to the range of experimental values.

The small frictional force between the two surfaces in contact directly affects the accuracy of the gravity acceleration value and is a source of systematic error, which is why the line of best fit was deviated from the origin.

It is also observed the influence of errors in taking measurements due to the delay time that exists in the human being between activating and deactivating the acceleration reading, thus showing a deviation in the measurements as a random error of the experimentation.

For further research, in order to reduce errors and improve the didactic methodological strategy, the following recommendations shown in Table 3, below, should be considered:

Table 3. *Table of weaknesses and recommendations designed by the Author*

Weaknesses	Recommendations
Friction between contact surfaces.	- Friction could be further reduced by polishing the surfaces and then placing the oil on both surfaces.
Reading errors in acceleration measurements.	- A simple experiment could be used to measure a person's reaction time https://creandoconciencia.org.ar/enciclopedia/conduccion-racional/reaccion-y-control/medicion-del-tiempo-de-reaccion.pdf and select the person with the shortest reaction time.

As for the strengths of the experiment, the measurement of the tilt angle using the other smartphone. The proposed didactic methodological strategy yields gravity acceleration values very close to the real value.

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